Design of Mechanical Power Transmissions (MEC8029) Individual Assignment	Newcastle University UK | Malaysia | Singapore
Electric vehicle gearbox design (2021-22)	NEWCASTLE UNIVERSITY School of Engineering

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1. Introduction

This report aims to design and optimise a single stage parallel axis gear pair for use by electric vehicle manufacturers to retrofit to cars and similar modes of transport. The analysis will be done using the Dontyne Systems Gear Production Suit software using an iterative analysis process. The table below is a brief technical specification highlighting the minimum requirements of the system.

Power (kW)	Rotations per Minute (RPM)	Application Factor (K_a)	Torque (Nm)		Desired Gear Ratio	Running Time (hrs)
			Pinion	Wheel		
95	5000	1.4	181.4	245.8	1.36	12000

Table 1.1 Technical Specification

The objectives of the report and analysis are as follows:

- Produce a concise report documenting the design and optimisation process used to derive the gear pairs geometry.
- Select appropriate macro-geometry based on the technical specification.
- Minimise the transmission error of the gear pair by specifying appropriate calculated micro-geometries.
- Simulate micro-geometries in using a tooth contact analysis (TCA) package.
- Maximise the power density of the gear pair by designing gears over the safety factors at a minimum volume.
- Ensure the design is accurate, reliable, quiet, relatively cheap, and efficient.

These objectives will be met by:

- Learning the fundamentals of gear design and specification through lecture material and relevant standards
- The above will be done before any simulation is carried out
- Understand how the major variables influence each other through an iterative simulation process
- Keep a concise logbook recording the parameters produced through each iteration and compare gear designs.

2. Gear Macro-Geometry

Selection of the macro-geometry for the power-dense gear box required careful consideration to all the major variables used to fully define a gear and how these variables influenced one-another. An iterative process was used to design the gear pairs to a minimum mass and volume whilst meeting Hertzian contact stress and bending stress safety factors.

The gear geometry was designed based on the torque, speed and life-requirements described in table 1 using an ISO 6336 stress analysis.

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Below shows a table summarising the gear pairs macro-geometry:

ISO Symbol		Macro-Geometry	Value		Units
Pinion	*Wheel*		*Pinion*	*Wheel*	
z_1	z_2	Number of teeth	31	42	
u_i		Desired Gear Ratio	1.36		
u		Gear Ratio	1.355		
m_n		Normal Module	3.5		[mm]
b		Facewidth	60		[mm]
d_1	d_2	Reference/Pitch Diameter	117.02	158.54	[mm]
d_{b1}	d_{b2}	Base Diameter	108.93	147.58	[mm]
d_{a1}	d_{a2}	Tip Diameter	124.19	165.38	[mm]
d_{f1}	d_{f2}	Root Diameter	108.44	149.63	[mm]
h_1	h_2	Tooth Depth	7.875	7.875	[mm]
α_n		Normal Pressure Angle	20		[Degrees]
β		Reference Helix Angle	22		[Degrees]
		Helix Hand	Left	Right	
x		Profile Shift Coefficient	0.024	-0.024	
j_n		Normal Backlash	0.2		[mm]

Table 2. Table Summarising Gear Pair Macro-Geometry

Macro Geometry Selection Methodology:

The bulleted list below highlights the process used to derive the gear pairs macro-geometry.

- Gear tooth number was selected based on the 1.36 gear ratio.
- Power, speed (RPM), life cycle and application factor were inputted into the GPS software.
- First iteration gear material and manufacturing data were inputted.
- Gear normal transverse modulus was selected to bring the stress safety factors above unity.
- Gear facewidth was chosen.
- Reference Helix angle was chosen
- Tooth normal backlash was chosen.
- Profile shift coefficient was chosen.

Justification of Macro-Geometry

Below is a bulleted list defending and justifying the gear pairs macro-geometry selection.

- The pinion tooth number z_1 was chosen to be 27. This value was chosen because of the requirements of the gear pair to be very quiet and high speed. Pinions with a tooth number of 23 + are recommended for such application. A prime number was chosen to minimise the hunting tooth frequency by having the highest common factor of unity. The wheel tooth number was calculated based of the equation $z_2 = u \times z_1$.
- The normal module was chosen through an iterative process comparing the mass and volume of the gear pair with the stress safety factors. A value of 3.5 proved to generate the lowest mass with a sufficient hertzian contact safety factor, designing for the bending safety factor was of less concern.
- The gears facewidth initial value was chosen to be 80 mm based of the equation $0.4 < b/d < 0.9$. Increasing the facewidth didn't positively influence the safety factor

enough to account for the large increase of mass resultant from the increased facewidth so after trial and error of various values, 60 mm was decided to be optimum.

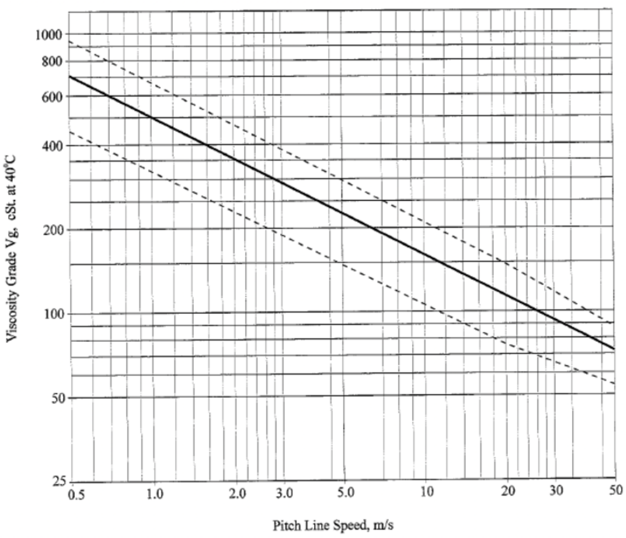
- The normal pressure angle of 20 degrees was chosen base of the ISO 53 standard demonstrating a basic Type D rack.
- A reference helix angle of 22 degrees was chosen because it generated an integer value of 2 for the overlap ratio $\varepsilon_{\boldsymbol{\beta}}$. An angle of 34 degrees also created an integer overlap ratio of value 3 but this value would have increased the elastic tooth deflection from 2.06μm to 3.03μm. This increase in helix angle would have also caused an increase in mass of 3.57 kg of the gear pair and an increased axial force of 618N experienced by the supporting bearings.
- A profile shift coefficient of ±0.024mm was applied to the pinion and wheel respectively. This value was calculated by the Dontyne Systems software as the value that balanced the bending stress safety factor.
- A normal backlash value of 2 mm was decided based of the recommended range of 0.1 – 0.3 mm from lecture material. Visual representation of the meshing gears supported that 0.2 mm was sufficient to prevent unwanted undercutting interactions.

Below is a table displaying values for some other notable system parameters.

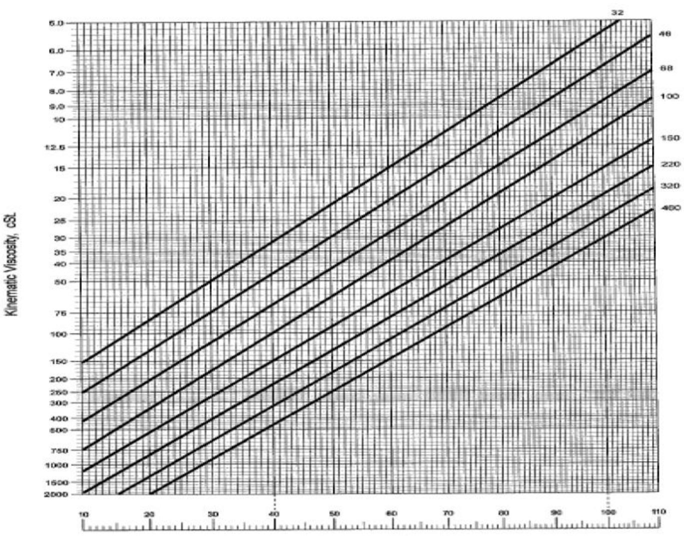
ISO Symbol		**Parameter Definition**	**Value**		**Units**
Pinion	*Wheel*		*Pinion*	*Wheel*	
m_1	m_2	Mass	5.06	9.24	[kg]
V_1	V_2	Volume	6.45e-4	1.18e-3	[m^3]
n_u		Viscosity	90		mm^2/s
		Basic Rack Tooth Profile	Type D		

Table 3. Additional Notable System Parameters

The viscosity value of 90 mm^2/s for the sprayed oil lubrication was found by applying the pitch line speed to the left graph to obtain the viscosity grade, then using this value on the right graph to estimate a suitable kinematic viscosity value to prevent scuffing due to metal on metal contact.



Graph 1. Viscosity Grade vs Pitch Line Speed [m/s]



Graph 2. Kinematic Viscosity vs Viscosity Grade

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3. Gear manufacturing method, geometry tolerances (ISO1328-1) and material and heat treatment (ISO 6336-5) specification.

Manufacturing Method and Material:

The power- dense gear pair is made from high quality, clean 20MoCr4 Steel that has been hobbed from a solid material block to the correct geometry, carburised and induction hardened. Following hardening, the gear will be sand blasted to increase compressive residual stresses and superfinished with finish ground methods to minimise surface roughness.

Property	Description
Gear Material	20MoCr4 Steel
Manufacturing Method	Hobbed -> Carburised -> Induction Hardened -> Sand Blasted -> Finish Ground
Material Quality	MQ
Grade	6
Case Depth @ 550 HV	0.75
Hardness [HV]	615
Bending Stress Number [MPa]	369
Contact Stress Number [MPa]	1215

Table 4. Gear Material Properties and Manufacturing Methodology

Geometry Tolerances:

The tolerances of the grade 6 gears major microgeometries are as follows:

Geometry	Tolerance		Units
	Pinion	Wheel	
Total Radial Composite Deviation	36	43	[µm]
Total Cumulative Pitch Deviation	27	35	[µm]
Total Tangential Component Deviation	38	47	[µm]
Profile Form Deviation	8.5	9.5	[µm]
Profile Slope Deviation	7.0	8.0	[µm]
Total Profile Deviation	11	13	[µm]
Helix Form Deviation	10	10	[µm]
Helix Slope Deviation	10	10	[µm]
Total Helix Deviation	14	15	[µm]

Table 5. Gear Geometry Tolerances

4. Micro-geometry specification

The micro-geometry of the power-dense gear pair was carefully chosen through calculations which will be discussed, and the corresponding transmission error will be simulated through the tooth contact analysis (TCA) package within the Dontyne Systems software. The table below highlights relative parameters associated with micro-geometry changes.

ISO Symbol	Geometry Description	Value	Unit
C_a	Tip relief	28.5	µm
C_b	Helix Crowning	1.00	µm
SAP	Start of Active Profile Diameter	119	mm
EAP	End of Active Profile Diameter	160.6	mm
∂	Elastic Tooth Deflection	2.06	µm

Table 6. Tooth Micro-Geometry Modifications

The amount of transmission error (TE) is directly related to the level of noise, vibration, and harshness (NVH) experienced by the gear pair. TE can be reduced by applying changed to the micro-geometry of the gear teeth to improve the smoothness of power transmission. Three applications of micro-geometry modifications have been applied including:

<u>Tip relief –</u>

Sufficient tip relief reduces the possibility of teeth prematurely meshing with one-another, producing shock loads which can heavily damage gears and cause premature failure. The quantity of tip relief applied to each gear was estimated of the mean mesh deflection ∂ plus the cumulative pitch/profile tolerance (the larger value is used).

$$\partial = \frac{F_t\ [N]}{b\ [mm] \times\ C_\gamma [N/mm/\mu m] \times\ \varepsilon_\alpha}$$

where F_t = Tangential Force [N], b = facewidth [mm], C_γ = Combined Mesh Stiffness [N/mm/µm] and ε_α = Transverse Contact Ratio.

$$\partial = \frac{3100}{60\ \times 16.5 \times 1.521}\ =\ 2.06\ \mu m$$

The cumulative pitch error tolerance value of 27µm was used over the cumulative profile error tolerance value of 13µm because it was a larger value and added a factor of safety.

Amount of Tip Relief = Mean mesh deflection + pitch error tolerance, giving a tip relief value of

$$C_a\ =\ 2.06\ \mu m\ +\ 27\ \mu m\ =\ 29.6\ \mu m$$

The long tip relief strategy of a parabolic nature was implemented meaning the tip relief begins at the highest point of single tooth contact (HPSTC) at a diameter of 119.09 mm and 160.52 mm for the pinion and wheel respectively. It was not necessary to add root relief providing tip relief has been considered for both gears as above.

<u>Helix Crowning -</u>

A helix crowning value of 1µm was applied only to the pinion to improve the load distribution of forces between meshing teeth. This value was selected through an iterative trial and error process and was chosen because it resulted in the lowest TE.

<u>End Relief –</u>

An end relief value of 0.5µm was chosen at either extremity of the pinion gear tooth starting 27mm either side from the centre line of the gear in an effort to mitigate the possibility of unwanted shock loading of teeth due to manufacturing or shaft misalignment. A similar iterative trial and error process as helix crowning value optimisation was used to derive the end relief quantity that minimised the TE value.

The resulting transmission error estimated using the Dontyne Systems TCA two-dimensional simulation amounted to 0.08µm after the above micro-geometry changes were implemented as shown in image 1.

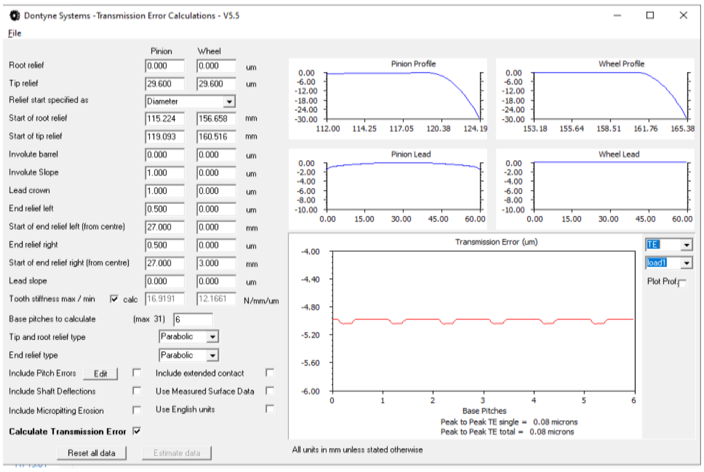

Image 1. Transmission Error TCA Calculations

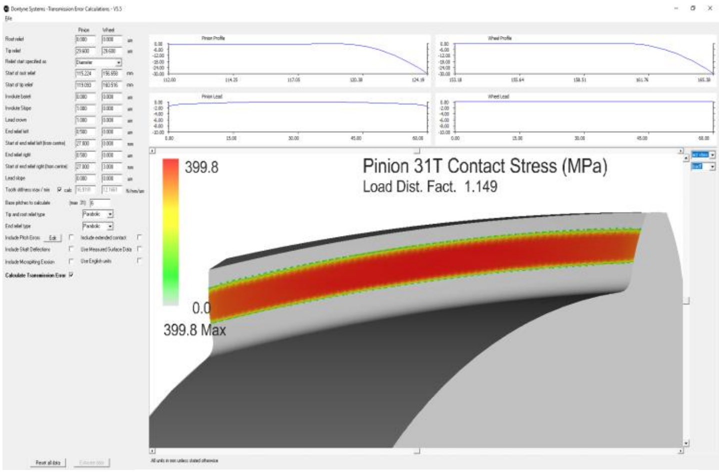

Image 2. Contact Stress Face Load Distribution

The software also generates a colourmap image of the contact stress intensity distribution throughout the profile of the gear tooth as shown in image 2. The chosen parameters ensure load is evenly distributed throughout the flank of the tooth, reducing the chances of micro-pitting and scuffing.

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5) ISO 6336 bending and contact stress fatigue analysis using Dontyne GPS software.

The Dontyne Systems GPS software allows the user to perform fatigue stress analysis on gear pairs to a very high accuracy based on complex formula. Below is a report generated from the software in the standardised Design Unit Format:

```
*************************************  pinion material IF
*           DONTYNE SYSTEMS         *  wheel material  IF
*        www.dontynesystems.com     *  hardness        HV      615     615
*************************************  material quality        MQ      MQ
* (University of Newcastle Upon Tyne *  roughns flnk/µm Rz      6.0     6.0
*  Design Unit Report Style Format)  *  roughns root/µm Rz      15.0    15.0
*************************************  viscosity @ 40C nu          90
* Dontyne Systems ISO 6336 Rating    *  pitting permitted?      no      no
* V 5.60 Bld74 1/23/2022 9:22:09 PM * reversing duty?         no      no
* Final_vary_for_report              *
*************************************  applicatn factr KA          1.400
number of Teeth z      31      42      required life/h             12000
normal module   mn         3.500       load cycles     NL      3.6e+09 2.7e+09
transvrs module mt         3.775       mesh power/kW   P           95.00
gear ratio      u          1.355       torque/Nm       T       181.4   245.8
centres         a          137.78      tr tang force/N Ft          3100.9
facewidth       b      60.00   60.00   speed/RPM       n       5000.0  3690.5
reference diam  d      117.02  158.54  pitch line speed/m/s        30.6
base diameter   db     108.93  147.58  tip relief/µm   Ca          28.5
tip diameter    da     124.19  165.38
root diameter   df     108.44  149.63  helix modifictn none
tooth depth     h      7.875   7.875   fav contact pattn posn verifctn?  no
internal diameter      0.00    0.00    wheel web thickness
norm pres angle alphan     20.000      pinion offset   s           0.000
transv pres ang alphat     21.433
wkng tr pr ang  alphawt    21.433
ref helix angle beta       22.000
base helix ang  betab      20.611
prof shift coef x      0.024   -0.024  hx dev elast/µm fsh         0.2
sum of "" coefs Sigx       -0.000      quality grade   q       6       6
bsc rack dedend hfP/mn 1.250   1.250   hx dev manuf/µm fma         39.1
bsc rk root rd rofP/mn 0.390   0.390   init'l misal/µm Fbetax      39.4
residual protub Spr/mn 0.000   0.000   run-in misal/µm Fbetay      33.5
root chord lgth sFn/mn 2.101   2.138   stiff(N/mm/µm)  cgamma      16.5
bending mom arm hFa/mn 0.921   0.953
root radius     roF/mn 0.533   0.525
tr contct ratio epsalpha   1.521       overlap ratio   epsbeta     2.044
-----------------------------------FACTORS-----------------------------------
resonance ratio N          0.700       dynamic fct     KvB         1.515
------------TOOTH FLANK---------------  --------------TOOTH ROOT-------------
face load factr KHbeta     2.927       face load factr KFbeta 2.547   2.547
transv load fct KHalpha    1.118       transv load fct KFalpha    1.118
zone factor     ZH         2.346       form factor     YF      0.956   0.955
elasticity fctr ZE         190.272     stress conc fct YS      2.032   2.050
single pair fct ZB/ZD  1.095   1.095   notch parameter qs      1.971   2.035
cont ratio fctr Zepsilon   0.811       cont ratio fctr Yepsilon   0.682
helix ang factr Zbeta      1.039       helix ang factr Ybeta      1.025
life factor     ZN     0.877   0.885   life factor     YN      0.868   0.873
lub inf fct     ZLZVZR 0.943   0.943   notch sensy fct YdrelT 0.995   0.995
work-hardng fct ZW         1.000       surface factor  YRrelT 0.976   0.976
size factor     ZX         1.000       size factor     YX      1.000   1.000
-------CONTACT STRESS (N/mm²)---------  -------BENDING STRESS (N/mm²)---------
allw stress num sgHlm  1289.7  1289.7  allw stress num sigFE  738.0   738.0
permiss stress  sigHP  1094.9  1104.3  permiss stress  sigFP  621.6   625.8
"" (reference)  " ref  1145.8  1145.8  " (reference)  " ref   716.5   717.0
contact stress  sigH   882.4   882.4   root stress     sigF   177.6   178.8
safety factor   SH     1.24    1.25    safety factor   SF     3.50    3.50
min safety fctr SHmin      1.00        min safety fctr SFmin      1.00

Angles are in ° and distances in mm unless otherwise stated.
```

Image 3. Dontyne Systems Design Unit Format Report

Resulting safety factors:

Below is a table highlighting the contact and bending stress safety factors as calculated by the Dontyne GPS Software.

Safety Factor	Pinion	Wheel
Hertzian Contact Stress S_H	1.24	1.25
Bending Stress S_F	3.50	3.50

Table 6. Factors of Safety

As seen above, the designed gear pair has a contact stress safety factor of 1.24 and 1.25 for the pinion and wheel respectively. This would be a relatively low safety factor if these gears were to be produced by the thousands for a commercial application, however, many factors that have been considered within this analysis have amplified the load drastically such as $K_{H\beta}, K_A, K_v$ acting as prior safety measures. For this reason, the aforementioned safety factors will be sufficient enough to allow the gear pair to operate successfully even if they were to constantly run at the full 95kW power rating.

The most challenging element of this analysis was to lower the bending safety factor of 3.50 to a value closer to the contact safety factor around 1.24/1.25. Some reduction was achieved by changing the material type from through hardened steel to flame or inductions hardened steel and increasing the surface hardness value to 615 HV. The assumption was made that after many geometry change iterations the discrepancy between the two safety factors was inevitable due to the loading conditions and desired ratio.

Misalignment Values $F_{\beta x}$:

Three main misalignments were taken into consideration for this gear pair optimisation analysis including:

- Gear Manufacturing Error Misalignment f_{ma}
- Gearbox Case Misalignment
- Shaft Deflection Misalignment f_{sh}

Gear Manufacturing Error f_{ma} –

This value f_{ma} was calculated by rooting the sum of the squares of the pinion and wheel helix slope deviations as shown in the below equation:

$$f_{ma} = \sqrt{f_{h\beta 1}^2 + f_{h\beta 2}^2} = \sqrt{10^2 + 10^2} = 14.14\ \mu m$$

Gearbox Case Misalignment –

According to the provided technical specification, the 5-axes CNC milling machine used to manufacture the gearbox housing has a tolerance of ±25µm. Theoretically, the misalignment could be + 50mm considering a worst-case scenario at either side of the shaft (2 multiplied by 25µm) but this case would be very unlikely. This scenario was considered, and a pair of gears were designed to meet the multiplied loads due to the higher $F_{\beta x}$ value. Considering +50 µm increased the load 3.519 times through a face contact load distribution factor $K_{H\beta}$ = 3.519, the gear pair designed that was capable of withstand these loads seemed too heavy and large to be practical for the given technical specification.

The manufacturing error used in the final analysis was 25µm (from the CNC machine) plus 14.14 µm (manufacturing error of the gears) resulting in a cumulative f_{ma} of 39.14 µm. This gave a face load distribution factor $K_{H\beta}$ = 2.927.

Shaft Deflection Misalignment f_{sh}–

The shaft deflection analysis tool within the Dontyne Systems GPS was used to estimate the shafts misalignment from load deflections f_{sh} with the analysis setup as shown below:

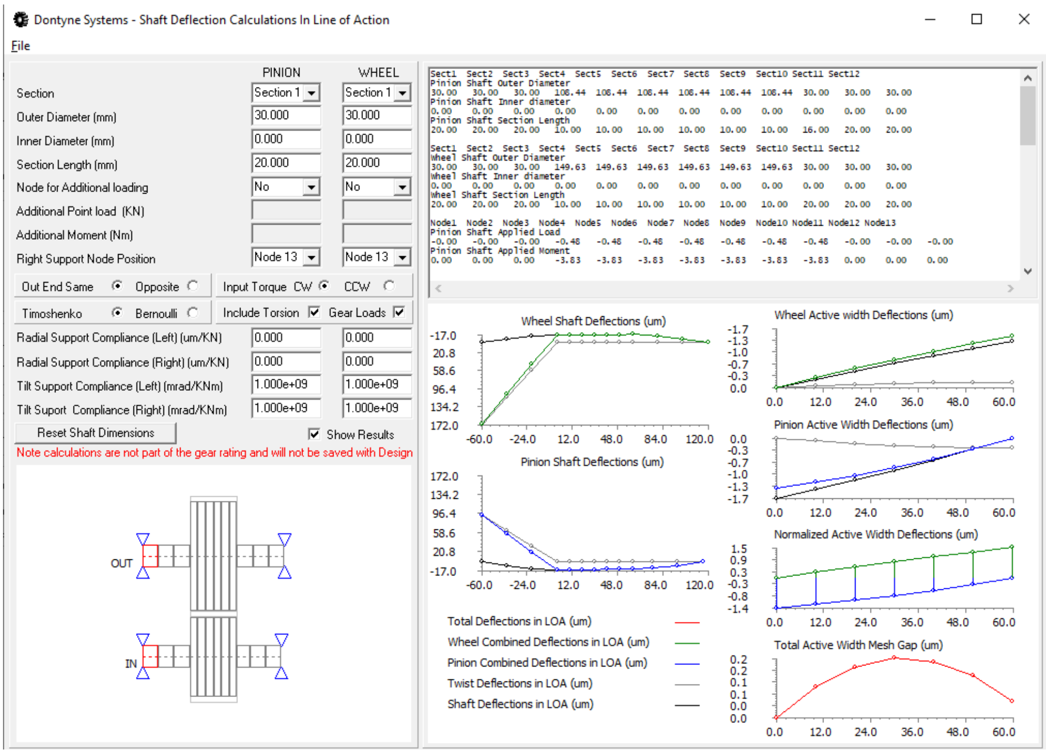

Image 4. Shaft Misalignment from Loading

Both the input and output shafts have a uniform diameter of 30 mm along with every sector having a length of 20 mm apart from sector 10 on the input pinion shaft, holding a length value of 20 mm. The reason for this is because if the pinion gear is beyond the shaft's midpoint slightly towards the opposite end of the input torque, the shaft shear stress and torsional stresses cancel each other out minimising shaft deflection. The torque is input in the clockwise direction causing the input shafts axial force to acts to the left due to the left-hand helix configuration. For the wheel, the torque is output on the same side of the torque input but in an anticlockwise direction, with the wheels axial force acting towards the right.

The overall misalignment due to the forces acting on the shaft was $f_{sh} = 0.2\ \mu m$ and was considered for the design and optimisation.

Discussion of Load Factors -

Dynamic Factor (K_v): The dynamic loading factor takes account for self-excited loads and the variation of tooth deflection caused by the variation in the number of teeth engaged at one time, resulting in a change of mesh stiffness. The Dontyne GPS Software has calculated a value of K_v = 1.514 effectively multiplying the load experienced during mesh by a factor of 1.514. The dynamic load factor can pose an issue when the gears are preforming at low loads causing the gears to rattle which is a common problem for automotive gears.

Face Load Distribution Factor for Hertzian Contact Stress ($K_{H\beta}$) and Bending Stress ($K_{F\beta}$): These face load distribution factors consider the uneven distribution of forces across the face with due to misalignment due to manufacturing errors resulting from the manufacturing process, shaft misalignment due to force endued shaft deflections and gearbox misalignments. This factor also considered the beneficial influence on micro-geometrical changes to the gear tooth such as helix crowning and tip relief. The $K_{H\beta}$ factor influences the load of the contact stress and the $K_{F\beta}$ factor influences the bending stress.

6) Gearbox cross section

Below shows a hand drawn cross section of the designed gear box with all components suitably constrained axially and radially. The pinion is a left-hand helix and with the torque being applied in the anti-clockwise direction, resulting in the axial force acting towards the top of the page. This axial force travels through the deep groove ball bearing and is reacted by the end cap.

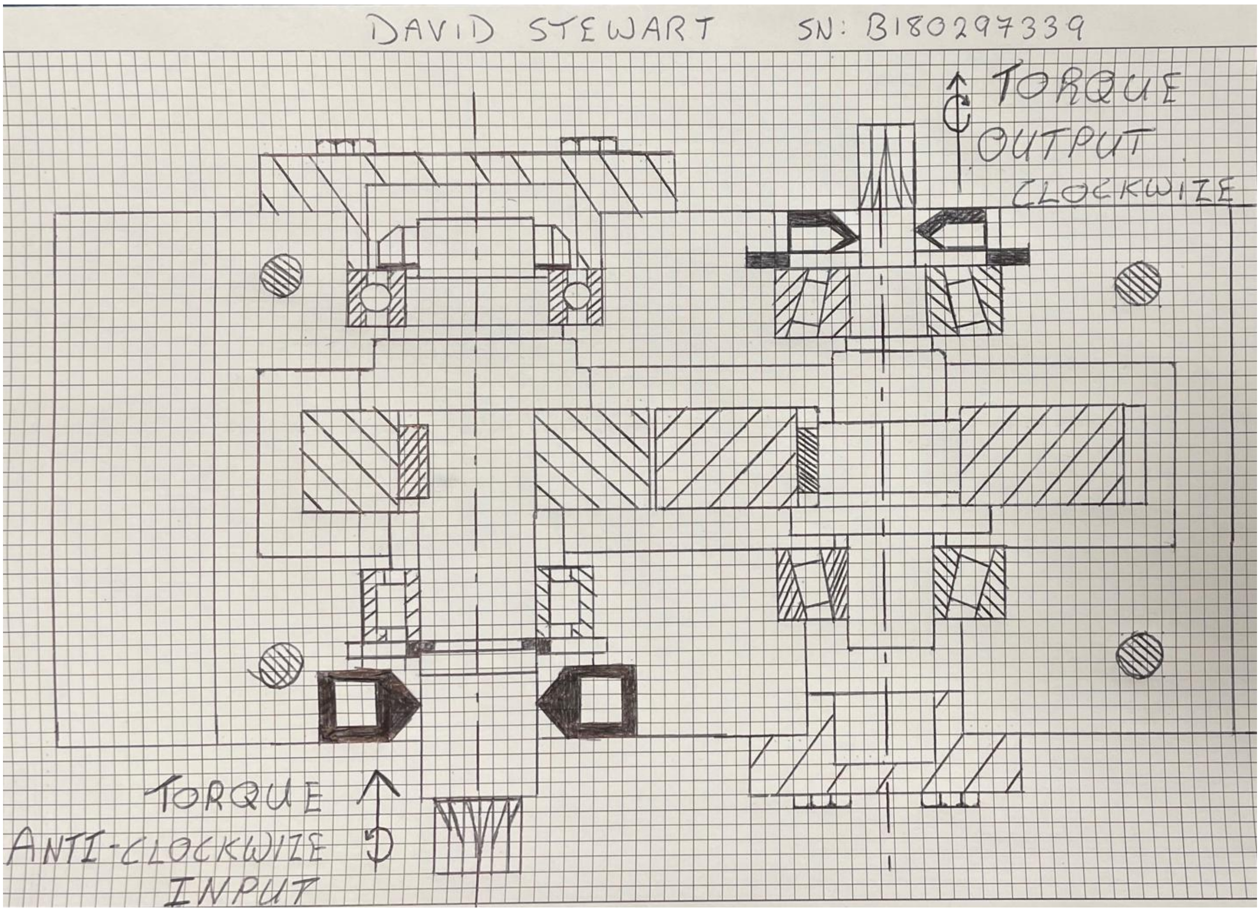

Image 5. Hand Drawn Cross Section of Gear Box

Considering the driven wheel, the axial force acts with an equal magnitude in the opposite direction to the pinions axial force. This force travels through the bottom taper bearing onto the shoulder of the housing where it is dissipated through the housing vibrations and heat.

7) Discussion and Conclusions

A single stage parallel axes gear pair has been designed and optimised using an iterative analysis process executed on Dontyne Systems Gear Production Suit. Macro-geometry was chosen based on the given gear ratio, power input, application factor and life cycle with the aim to produce power dense, low mass, high accuracy reliable gears for a high-speed application running with minimum noise, vibration, and harshness.

The bullet points below highlight some of the strong features within the design along with some areas needing attention:

Advantages –

- Minimum Mass of 14.3kg.
- Theoretically very quiet gears with an integer overlap ratio.
- Solid factors of safety after consideration of many loading factors $K_{H\beta}, K_A, K_v$.
- Gearbox occupies a small volume.
- Large power density.
- High Quality materials make for an accurate and reliable gear system.

Disadvantages –

- Factor of safety for bending stress is larger than necessary resulting in gears of a mass heavier than needed.
- High Quality material/grades and long manufacturing route may result in the gears being too expensive to implement commercially.
- Gear pair is approaching the resonance ratio (N=0.7) which may result in destructive self-excited vibrations damaging the gearbox.

To conclude, a parallel axis single stage gear box has been designed to last 12000 hours, rotating at 5000 rotations per minute with a power rating of 95kW. This report fully defines the parameters of the gear pair both micro and macro-geometries, and the analysis process taken to design and optimise the system using the advanced simulation software Donytne Systems Gear Production Suit.

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